APPLYING THE NASH BARGAINING SOLUTION FOR A REASONABLE ROYALTY

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I. Introduction 148
II. Elements of a Licensing Bargain 152
A. Operating Revenue 153
B. Operating Cost 153
C. Operating Income 153
D. Royalty 153
E. Disagreement Payoffs 154
F. Bargaining Weight 155
III. The Asymmetric Nash Bargaining Solution 155
IV. Normalized Royalty Model 158
V. Estimation of the Bargaining Weight 159
A. The Original Nash Bargaining Solution 161
VI. Discussion 162
A. Estimation of Bargaining Strengths Independent of the Disagreement Payoffs 162
Number of Competitors as Strength 162
Market Share as Strength 163
Life of the Patent as Strength 164

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Example 164
B. Estimation of Bargaining Strengths Using Disagreement Payoffs 165
C. Estimation of Bargaining Strength Using Combinations 170
1. Solutions That Violate Pareto Efficiency 171
VII. Nomographs 171
VIII. Conclusion 173

I. INTRODUCTION

Following a finding that a firm has infringed upon the patent of another, the patentee is entitled to damages that will compensate for any infringement and that compensation cannot be less than a "reasonable royalty for the use made of the invention by the infringer."[1] In essence, this statute sets a minimum for what the patentee can recover, which is the royalty that the patentee and the infringer would have actually agreed upon had there been no infringement suit. If a patentee cannot prove the actual damages for any or all of the infringing sale, then a reasonable royalty is calculated and awarded.[2] To date, the courts have utilized two different approaches for calculating "reasonable royalty" damages, the *Georgia-Pacific* approach and the Analytical approach. This Article focuses on the former and discusses how its full potential is not being realized.

John Forbes Nash studied cooperative bargaining and created a solution, known as the Nash Bargaining Solution (NBS). NBS is a unique solution to a two-person

[1] 35 U.S.C. § 284.
[2] *See id.*

bargaining problem that satisfies the axioms of scale invariance, symmetry, efficiency, and independence of irrelevant alternatives.[3] The NBS has been applied to economics, game theory, and employment.[4]

While there is potential for NBS to aid courts' determination of "reasonably royalty" damages, there has been limited success applying the NBS when assigning intellectual property damages due to the difficulty of relating it to the specific facts of the case. Because of this, parties are not taking advantage of *Georgia-Pacific* factor fifteen. This Article intends to clarify the NBS so that it can be applied to the facts of a case. This Article normalizes the NBS and provides a methodology for determining the bargaining weight in Nash's solution. Several examples demonstrate this normalized form, and a nomograph is added for computational ease.

In U.S. patent litigation, there are two predominant ways to compensate a licensor when a firm infringes on its intellectual property. One way is to calculate the profit that was lost due to the infringement.[5] The other way is to designate a reasonable royalty.[6] A reasonable royalty is defined as a royalty assigned to the licensor for its

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[3] John F. Nash, *The Bargaining Problem*, 18 ECONOMETRICA: JOURNAL OF THE ECONOMETRIC SOCIETY 155, 155–62 (1950); John Nash, *Two-Person Cooperative Games*, 21 ECONOMETRICA: JOURNAL OF THE ECONOMETRIC SOCIETY 128, 128–140 (1953).

[4] *See*, *e.g.*, Ken Binmore et al. *The Nash Bargaining Solution in Economic Modelling* 17 THE RAND JOURNAL OF ECONOMICS 176 (1986) (applying NAS to economics); Pierre Cahuc et al. *Wage Bargaining with On-the-Job Search: Theory and Evidence* 74 ECONOMETRICA 323 (2006).

[5] Nancy J. Linck & Barry P. Golob, *Patent Damages: The Basics*, 34 IDEA 13, 13-14 (1993).

[6] *Id.*

intellectual property by the licensee that is fair to both parties.[7]

Assigning a reasonable royalty is especially difficult in a dispute situation because of the difficulty for an arbiter or court to attribute a royalty that is perceived as fair for both parties. A famous District Court case *Georgia-Pacific vs. United States Plywood Corp*[8] demonstrated the complexity of assigning a reasonable royalty in litigation involving patents. As a result of the case, the District Court established fifteen guidelines for determining a reasonable royalty.[9] Notably, guideline fifteen allows for a hypothetical license negotiation when the infringement began.[10] This guideline implies that the NBS can be used as a justification for assigning a reasonable royalty because the NBS is based on two rational parties cooperatively bargaining to increase their profit over and above their opportunity costs by partitioning the surplus profit based on the value they each bring to the agreement.

In recent court cases, some judges have steered clear from using the NBS because parties often do not

[7] *Id.* at 21–22; Jarosz, John C. Jarosz & Michael J. Chapman, *The Hypothetical Negotiation and Reasonable Royalty Damages: The Tail Wagging the Dog*, 16 STAN. TECH. L. REV. 769, 774–75 (2013).

[8] Georgia-Pacific Corp. v. U.S. Plywood Corp., 318 F. Supp. 1116, 1120 (S.D.N.Y. 1970), *mod. and aff'd*, 446 F.2d 295 (2d Cir. 1971), cert. denied, 404 U.S. 870 (1971).

[9] *Id.*

[10] *Id.* (Guideline fifteen is: "The amount that a licensor (such as the patentee) and a licensee (such as the infringer) would have agreed upon (at the time the infringement began) if both had been reasonably and voluntarily trying to reach an agreement; that is, the amount which a prudent licensee -- who desired, as a business proposition, to obtain a license to manufacture and sell a particular article embodying the patented invention -- would have been willing to pay as a royalty and yet be able to make a reasonable profit and which amount would have been acceptable by a prudent patentee who was willing to grant a license.").

apply it to the specific facts of the case.[11] As a result, judges often criticize the NBS solution when determining a reasonable royalty.[12] Because Nash's solution is often not tailored to the specific facts of the case, parties are not taking full advantage of *Georgia-Pacific* guideline fifteen. Another reason for criticism is the NBS is not simple to calculate or easy to interpret, so it is difficult for a jury or court to apply.[13] To demystify the NBS, this Article introduces certain normalizations that provide for a simple calculation of damages. These normalizations make the NBS a powerful tool to value intellectual property and provide guidance in assigning proper compensation.

First, this Article applies Nash's solution in a more business-friendly manner by using terminology common in financial statements. Additionally, this Article normalizes each monetary term in the NBS by the operating income. By doing this, the parties can better interpret the NBS and do not need to know exact dollar amounts when determining a royalty. The Choi and

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[11] *See e.g.,* VirnetX Inc. v. Cisco Systems, Inc., 767 F.3d 1308, 1325–26 (Fed. Cir. 2014); Oracle Am., Inc. v. Google Inc., 798 F. Supp. 2d 1111, 1120 (N.D. Cal. 2011); Suffolk Techs. LLC v. AOL Inc., No. 1:12cv625, 2013 U.S. Dist. LEXIS 64630, at *4–*5 (E.D. Va. Apr. 12, 2013); Limelight Networks, Inc. v. Xo Commc'ns., LLC Civil Action No. 3:15-CV-720-JAG, 2018 U.S. Dist. LEXIS 17802, at *7–*8 (E.D. Va. Feb. 2, 2018).

[12] Lance Wyatt, *Keeping Up with the Game: The Use of the Nash Bargaining Solution in Patent Infringement Cases*, 31 SANTA CLARA COMP. & HIGH TECH. L. J. 427, 446–47 (2015); J. Gregory Sidak, *Bargaining Power and Patent Damages*, 19 STAN. TECH. L. REV. 1, 4–5 (2015); Zelin Yang, *Damaging Royalties: An Overview of Reasonable Royalty Damages*, 29 BERKELEY TECH. L. J. 647, 662–64 (2014).

[13] Wyatt, *supra* note 9, at 430–31 (internal citations omitted) ("First, damages experts often use the NBS improperly, failing to apply the specific facts of the case to their calculations. Second, damages experts typically fail to adequately explain the NBS to courts and juries.").

Weinstein[14] Two Supplier World (TSW) model is the basis for these authors' modifications.

Second, Nash's original solution assigns equal bargaining strength to each party. However, this equal bargaining strength assumption is generally not realistic.[15] This Article shows Nash's solution with an arbitrary bargaining weight to account for unequal bargaining strengths and presents a methodology for determining those strengths.

Third, a nomograph of the NBS is supplied to make it easy for parties to obtain a reasonable royalty using a simple straight edge graphically. Nomographs are useful to provide visualization so the NBS can be better explained.

By taking these steps, parties can take advantage of *Georgia-Pacific* factor fifteen by allowing the NBS to be tailored to the specific facts of the case. This Article attempts to clarify the use of the NBS, so the royalty assigned is both legally defensible and mutually beneficial.

II. Elements of a Licensing Bargain

This Article proposes the NBS be recast into a simple normalized form using common terms found on a financial statement to introduce common business terminology.[16] These terms are outlined below.

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[14] William Choi & Roy Weinstein, *An Analytical Solution to Reasonable Royalty Rate Calculations*, 41 IDEA 49, 58–60 (2001).

[15] Richard Higgins & Jeffrey Klenk, *An Application of Nash Bargaining to Intellectual Property Negotiations*, 25 FED. CIR. BAR J. 125, 128–29 (2015).

[16] *See* PASCAL QUIRY ET AL., CORPORATE FINANCE: THEORY AND PRACTICE 30, 32, 58, 152 (5th ed. 2018) (explaining common business terminology).

A. Operating Revenue

The operating revenue is the revenue generated from the intellectual property and is denoted by O_R. It does not include income from unusual events or income that is not primarily due to the use of the intellectual property.

B. Operating Cost

The operating cost is the expense associated with producing and selling the product incorporating the intellectual property. It is defined as O_C and does not include expenses from non-primary sources or unusual events.

C. Operating Income

The operating income, or profit, is determined by subtracting the operating cost from the operating revenue: $O_I = O_R - O_C$. In formulating the asymmetric NBS, the licensor and licensee's operating income are denoted by π_1 and π_2, respectively, where the total profit in the system is O_I .

Operating Margin

The operating margin, O_M , is operating income divided by operating revenue and is expressed as $O_M = O_I/O_R$.

D. Royalty

The royalty is what the licensee will pay the licensor to use the intellectual property.[17] There are two

[17] Linck & Golob, *supra* note 5, at 18; John C. Jarosz & Michael J. Chapman, *Application of Game Theory to Intellectual Property Royalty Negotiations*, *in* LICENSING BEST PRACTICES: STRATEGIC, TERRITORIAL,

common ways to calculate a royalty.[18] One way is assigning a royalty on each unit sold.[19] The other is obtaining a royalty based on a percentage of revenue[20] by multiplying the operating revenue with the royalty rate, r. In this Article, the focus is solely on a royalty based on revenue.

E. Disagreement Payoffs

A disagreement payoff is the opportunity cost of making the deal.[21] In other words, disagreement payoffs are profits from a hypothetical negotiation that did not happen but could have happened if the parties did not agree to a deal. Disagreement payoffs are typically expressed as monetary amounts and are represented in this paper by d_1 and d_2 for the licensor and licensee, respectively. However, for computational ease, the disagreement payoffs are normalized by the operating income, and these are expressed as $d_1^\dagger$ and $d_2^\dagger$ for the licensor and licensee, respectively. A normalized disagreement payoff equal to "1" implies a party is indifferent between making the deal and not making the deal since theparty could earn the same profit regardless. For emphasis, a normalized disagreement payoff of $d_2^\dagger = 0.5$ means the licensee's opportunity cost is half the total profit that a deal with the licensor can generate. Each parties' normalized disagreement payoffs can vary between zero and one. However, the sum of the normalized disagreement payoffs cannot exceed one, or a deal cannot be made since there is

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AND TECHNOLOGY ISSUES 241, 242 (Robert Goldscheider & Alan H. Gordon eds., 2006).

18 Linck & Golob, *supra* note 5, at 18; Jarosz & Chapman, *supra* note 18, at 242.

19 Linck & Golob, *supra* note 5, at 18.

20 Jarosz & Chapman, *supra* note 17, at 242.

21 *Id.* at 247

not enough profit to give each party their opportunity cost. The disagreement point is denoted by $d^{\dagger} = \left(d_1^{\dagger}, d_2^{\dagger}\right)$.

F. Bargaining Weight

A bargaining weight quantifies each party's influence in the negotiation and determines how the parties split the surplus from making the deal.[22] The licensor's bargaining weight is α, and the bargaining weight for the licensee is $1 - \alpha$, where the weight is between zero and one. Consequently, the party with the larger weight will obtain a larger surplus from making the deal. When applying the NBS, it has been common practice to assign each party a weight equal to 1/2, which implies that each party has the same influence in the negotiation.[23]

III. THE ASYMMETRIC NASH BARGAINING SOLUTION

John Nash developed the NBS, which provides a method for two parties who enter a profit-making agreement to optimally share those profits.[24] The axioms that satisfy the original NBS are:

1. **Individual rationality:** No party will agree to accept a payoff lower than the one guaranteed to it under disagreement.
2. **Pareto efficient:** None of the parties can be made better off without making at least one party worse off.

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[22] *Id.* at 248.
[23] *Id.* at 248; Mark A. Lemley & Carl Shapiro, *Patent Holdup and Royalty Stacking*, 85 TEX. L. REV. 1991, 1997 (2006); Jonathan D. Putnam & Andrew B. Tepperman, *Bargaining and the Construction of Economically Consistent Hypothetical License Negotiations*, 2004 LICENSING J. 8, 9.
[24] Nash, *The Bargaining Problem*, *supra* note 3, at 155–62; Nash, *Two-Person Cooperative Games*, *supra* note 3, at 128–40.

3. **Symmetry:** If the parties are indistinguishable, the agreement should not discriminate between them.
4. **Affine transformation invariance:** An affine transformation of the payoff and disagreement point should not alter the outcome of the bargaining process.
5. **Independence of irrelevant alternatives:** All threats the parties might make have beenaccounted for in the disagreement point.[25]

However, the introduction of a bargaining weight into the NBS allows the parties to be distinguishable when $d_1^{\dagger} = d_2^{\dagger}$ (potentially violating symmetry), known as the asymmetric NBS.[26] An excellent summary of the literature involving the asymmetric NBS and its use in intellectual property litigation is found in Bhattacharya.[27] The bargaining weight can be influenced by other forces or tactics employedby the parties, which can be independent of the disagreement payoffs. These forces should be accounted for because they ultimately affect how the surplus is divided.[28]

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[25] *Id.* at 247.

[26] ABHINAY MUTHOO, BARGAINING THEORY WITH APPLICATIONS 35 (1999).

[27] Rajeev R. Bhattacharya, *Nash Bargaining Solution and its Generalizations in Intellectual Property Litigation; VirnetX and Analysis of the Court's Decision*, 19 J. INT'L BUS. & L. 50, 58–60 (2019).

[28] MUTHOO, *supra* note 16 ("However, the outcome of a bargaining situation may be influenced by other forces (or, variables), such as the tactics employed by the bargainers, the procedure through which negotiations are conducted, the information structure and the players' discount rates. However, none of these forces seem to affect the two objects upon which the NBS is defined, [the disagreement payoffs], and yet it seems reasonable not to rule out the possibility that such forces may have a significant impact on the bargaining outcome.").

The asymmetric NBS is formed from the constrained maximization problem:[29]

$$\max_{\pi_1,\pi_2}(\pi_1 - d_1)^{\alpha}(\pi_2 - d_2)^{1-\alpha} \tag{1}$$

Subject to the following conditions:

$$\pi_1 \geq d_1 \tag{2}$$

$$\pi_2 \geq d_2 \tag{3}$$

$$\pi_1 + \pi_2 \leq O_I \tag{4}$$

Maximum occurs when:

$$(1-\alpha)\,(\pi_1^* - d_1) = \alpha\,(\pi_2^* - d_2) \tag{5}$$

$$\pi_1^* + \pi_2^* = O_I \tag{6}$$

Solving for the optimal partition of the profits gives the final result:

$$\pi_1^* = d_1 + \alpha(O_I - d_1 - d_2) \tag{7a}$$
$$\pi_2^* = d_2 + (1-\alpha)(O_I - d_1 - d_2) \tag{7b}$$

Eq. (7)'s interpretation is that the parties first agree to give each other their respective disagreement payoffs and split the remaining profit (surplus) according to their bargaining weight.

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[29] Ehud Kalai, *Nonsymmetric Nash Solutions and Replications of 2-Person Bargaining*, 6 INT'L J. GAME THEORY 129, 130-31 (1977); Ken Binmore, Ariel Rubinstein & Asher Wolinsky, *The Nash Bargaining Solution in Economic Modelling*, RAND J. ECON. 176 ,186 (1986); ALVIN E. ROTH, AXIOMATIC MODELS OF BARGAINING 19-20 (1979).

IV. NORMALIZED ROYALTY MODEL

To make the TSW model more practical, Eq. (7) was modified to introduce a royalty based on a percentage of operating revenue. Moreover, by simple algebraic manipulation, Eq. (7) can be modified where every monetary term is normalized by the operating income and varies between zero and one. Having each monetary term normalized is powerful because the parties do not need to think about specific dollar amounts. Instead, the parties can think in terms of fractions of profit.

The licensor is referred to as party 1 and the licensee as party 2. Under these assumptions, the payoffs for parties 1 and 2 are:

$$\frac{\pi_1^*}{o_I} = \frac{r\, o_R}{o_I} = \frac{r}{o_M} \tag{8}$$

$$\frac{\pi_2^*}{o_I} = \frac{o_R - o_C - r o_R}{o_I} = 1 - \frac{r}{o_M} \tag{9}$$

Additionally defining:

$$d_1^\dagger = \frac{d_1}{o_I} \quad 0 \leq d_1^\dagger \leq 1 \tag{10}$$

$$d_2^\dagger = \frac{d_2}{o_I} \quad 0 \leq d_2^\dagger \leq 1 \tag{11}$$

Substituting Eqs. (8), and (10)–(11) into Eq. (7a), the result for the optimal NBS is obtained with an arbitrary bargaining weight for party 1:

$$\frac{r}{o_M} = d_1^\dagger + \alpha\left(1 - d_1^\dagger - d_2^\dagger\right) \tag{12}$$

Where:

$$0 \leq d_1^\dagger + d_2^\dagger \leq 1 \tag{13}$$

To maintain Pareto efficiency, Eq. (12) must satisfy the following[30]:

$$\frac{\partial r}{\partial d_1^\dagger} > 0 \tag{14a}$$

$$\frac{\partial r}{\partial d_2^\dagger} < 0 \tag{14b}$$

The interpretation of Eq. (14) is that for a small positive change in party 1's disagreement payoff, the royalty should increase. In contrast, for a small positive change in party 2's disagreement payoff, the royalty should decrease - that is, a party cannot be made better off without making the other party worse off.

V. ESTIMATION OF THE BARGAINING WEIGHT

The bargaining weight, α, represents how the parties perceive their bargaining strength and how they see the other's bargaining strength. To account for all the perceptions of bargaining strength, the parameter, $P_{m,n}$ is introduced as party m's bargaining strength as perceived by party n, where m and n are variables that can only take on values of either 1 or 2. For example, $P_{1,2}$ is how the licensee (party 2) perceives the licensor's (party 1) bargaining strength.

Making the simple assumption that the bargaining strength of each party is the average of their perception and the perception of the other party, the following mathematical ansatz is introduced using two different equations to describe the bargaining weight of party 1:

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[30] Li Way Lee, *A Theory of Just Regulation*, 70 AM. ECON. REV. 848, 852 (1980).

$$\alpha_1 = \frac{1}{2}\left[P_{1,1} + P_{1,2}\right] \tag{15a}$$

$$\alpha_2 = 1 - \frac{1}{2}\left[P_{2,1} + P_{2,2}\right] \tag{15b}$$

Averaging Eqs. (15a)–(15b), the complete expression for the bargaining weight of party 1 is obtained:

$$\begin{aligned} \alpha &\equiv \frac{1}{2}[\alpha_1 + \alpha_2] \\ &= \frac{1}{2} + \frac{1}{4}\left[P_{1,1} + P_{1,2} - P_{2,1} - P_{2,2}\right] \qquad 0 \le P_{m,n} \le 1 \end{aligned} \tag{16}$$

Eq. (16) is critically important because a simple procedure now exists to define the bargaining weight of party 1. By formally defining the bargaining weight, each party's bargaining strengths can be incorporated to fit the particular facts of a case.

There are three basic approaches when calculating a bargaining weight. One approach is to treat α as a function independent of the disagreement payoffs. The second approach is to make the bargaining weight strictly a function of the disagreement payoffs. The third is a mixture of the first two approaches.

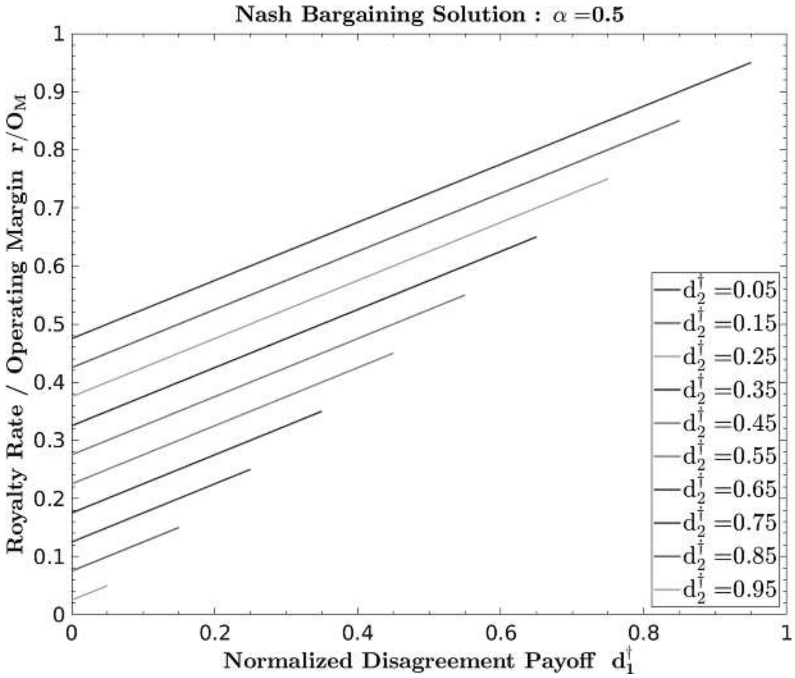

Figure 1: Family of Nash Bargaining Solutions Given Equal Bargaining Power

A. The Original Nash Bargaining Solution

When $P_{1,1} + P_{1,2} = P_{2,1} + P_{2,2}$ in Eq. (16), then $\alpha = 1/2$ and the original symmetric NBS is obtained, the following equation is formed:

$$\frac{r}{O_M} = d_1^\dagger + \frac{1}{2}\left(1 - d_1^\dagger - d_2^\dagger\right) = \frac{1}{2} + \frac{d_1^\dagger - d_2^\dagger}{2} \tag{17}$$

Fig. 1 presents the family of solutions of Eq. (17). Note that the lines of equal $d_2^\dagger$ are linear and equidistant from each other. Also, note that the lines are not the same length due to the constraint of Eq. (13).

VI. DISCUSSION

In this section, some hypothetical situations are presented to demonstrate the use of the NBS. Since the assignment of a party's perception of bargaining strength to a particular $P_{m,n}$ can be somewhat arbitrary, the examples given in this section are for illustration only. In the end, it is the job of the parties to provide a careful assessment of each of their perceptions and incorporate them appropriately into Eq. (16). By choosing these perceptions, the NBS can be applied to the specific facts of the case.

A. Estimation of Bargaining Strengths Independent of the Disagreement Payoffs

The use of Eq. (16) is demonstrated by a simple hypothetical negotiation involving bargaining strengths independent of the disagreement payoffs. Below is a discussion of the bargaining strengths number of competitors, market share, and life of the patent.

Number of Competitors as Strength

The bargaining strength of party 1 is "dependent on the relation between the hypothetical number of licensors and licensees in the market."[31] This is because if party 1 has a wide range of options to sell its intellectual property, then party 1 is presumably less concerned about making a deal with party 2. After all, the licensor can credibly walk away and license the technology to another firm. Therefore, if party 1 can sell its intellectual property to multiple licensees, the expectation is that party 1 has more bargaining strength. Conversely, if party 2 can license an acceptable

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[31] Sebastian Zimmeck, *A Game-Theoretic Model for Reasonable Royalty Calculation*, 22.2 ALB. L.J. SCI & TECH. 357, 405 (2011).

substitute, party 1's bargaining strength will diminish. The following equation is driven by the ratio of the number of licensors to the number of licensees in the relevant market.[32] The component of party 1's bargaining strength, as derived from the number of licensors and licensees in the market is:

$$P_{1,n}^{L} = 1 - \min\left[1, \frac{\text{Licensors}}{\text{Licensees}}\right] \tag{18}$$

The perception is assigned as $P_{1,n}$ because either party may perceive Eq. (18) as a component of party 1's bargaining strength.

Market Share as Strength

"In business, market share is regarded as the essential element of dominance."[33] As a result, valuing a component of party 1's bargaining strength by the amount of market share, s, is attractive instead of measuring potential profits. Using potential profits as a measurement of bargaining strength may not be appealing because profits are highly variable from year to year while market share is relatively constant over long periods of time. Additionally, courts often measure a firm's dominance by market share rather than profits.[34] Therefore, another measurement of bargaining strength is determining how much market share party 2 would gain as a result of the deal. The component of party 1's bargaining strength, as derived from market share, is:

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[32] *See id.* at 405.
[33] LI WAY LEE, INDUSTRIAL ORGANIZATION: MINDS, BODIES, AND EPIDEMICS 24 (2019).
[34] Duncan Cameron and Mark Glick, *Market Share and Market Power in Merger and Monopolization Cases* 17 MANAGERIAL AND DECISION ECON. 193, 193 (1996).

$$P_{1,n}^{S} = \frac{s}{S} \quad 0 \le s \le S \tag{19}$$

In Eq. (19), S denotes that fraction of the total market party 2 realistically desires.

Life of the Patent as Strength

Another perception of strength can be the time left until the patent expires. Presumably, party 1 is in a strong bargaining position when the patent is recently issued but is in a weak bargaining position when the patent is about to expire. Let the patent's life be denoted by T and the time elapsed since issue by t. The component of party 1's bargaining strength as derived from patent life is:

$$P_{1,n}^{T} = 1 - \frac{t}{T} \quad 0 \le t \le T \tag{20}$$

Example

In this hypothetical example, party 1 perceives its bargaining strengths with equal weight, the lack of acceptable substitutes for its patent, and the potential market share that the patent can bring to party 2. Party 2 perceives party 1's bargaining strength as only the life of the patent. Party 2 has a unique manufacturing base that can take full advantage of party 1's patent and perceives its bargaining strength as $P_{2,2} = 2/3$. Party 1 is aware of party 2's unique manufacturing capabilities, but only perceives party 2's strength as $P_{2,1} = 1/2$.

Substituting each perception into Eq. (16):

$$\alpha = \frac{1}{2} + \frac{1}{4}\left[\frac{P_{1,1}^{L} + P_{1,1}^{S}}{2} + P_{1,2}^{T} - \frac{1}{2} - \frac{2}{3}\right]$$

Eq. (21) can now be substituted into Eq. (12) to obtain the royalty for party 1.

B. Estimation of Bargaining Strengths Using Disagreement Payoffs

Disagreement payoffs can be a reasonable measure of bargaining strength because the parties can potentially walk away from the negotiation based on the disagreement payoffs alone.[35] Therefore, α can be a function of each party's disagreement payoff. This approach requires the least amount of information but requires the parties to determine a functional form of $\alpha\left(d_1^\dagger, d_2^\dagger\right)$ that adequately represents the negotiation. For a standard of fairness, it should be stipulated that when $d_1^\dagger = d_2^\dagger$, the parties should split the profit equally, which implies that symmetry is reintroduced. It is possible to construct an $\alpha\left(d_1^\dagger, d_2^\dagger\right)$ that reintroduces symmetry and yet provides variability in the bargaining weight.

Cases 1-3 in Table 1 are examples of symmetric bargaining weights driven by the parties' disagreement payoffs.

Case	P1,1	P1,2	P2,1	P2,2	$\alpha\left(d_1^\dagger, d_2^\dagger\right)$
1	$d_1^\dagger$	$d_1^\dagger$	$d_2^\dagger$	$d_2^\dagger$	$\frac{1}{2} + \frac{d_1^\dagger - d_2^\dagger}{2}$
2	$\frac{d_1^\dagger}{d_1^\dagger + d_2^\dagger}$	$\frac{d_1^\dagger}{d_1^\dagger + d_2^\dagger}$	$\frac{d_2^\dagger}{d_1^\dagger + d_2^\dagger}$	$\frac{d_2^\dagger}{d_1^\dagger + d_2^\dagger}$	$\frac{d_1^\dagger}{d_1^\dagger + d_2^\dagger}$
3	$\frac{d_1^\dagger}{d_1^\dagger + d_2^\dagger}$	$\frac{d_1^\dagger}{d_1^\dagger + d_2^\dagger}$	$\frac{1 - d_1^\dagger}{2 - d_1^\dagger - d_2^\dagger}$	$\frac{1 - d_1^\dagger}{2 - d_1^\dagger - d_2^\dagger}$	$\frac{d_1^{\dagger 2} + \left(2\, d_2^\dagger - 3\right) d_1^\dagger + d_2^{\dagger 2} - d_2^\dagger}{2\left(d_1^\dagger + d_2^\dagger\right)\left(-2 + d_1^\dagger + d_2^\dagger\right)}$

Table 1: Three Cases of Symmetric Disagreement Payoff Driven Bargaining Weights

[35] Jarosz & Chapman, *supra* note 17, at 262.

Case 1

In Case 1 of Table 1, each party assumes that its bargaining strength equals its disagreement payoff. Moreover, each party agrees that the other party's bargaining strength is its disagreement payoff. Remarkably, the resultant bargaining weight is the NBS of Eq. (17). Substituting Case 1 of Table 1 into Eq. (12):

$$\frac{r}{o_M} = \frac{{d_2^\dagger}^2 - {d_1^\dagger}^2 + 2\left(d_1^\dagger - d_2^\dagger\right) + 1}{2} \tag{22}$$

Eq. (22) shows a quadratic dependence on both $d_1^\dagger$ and $d_2^\dagger$, and this dependence is illustrated in Fig. 2.

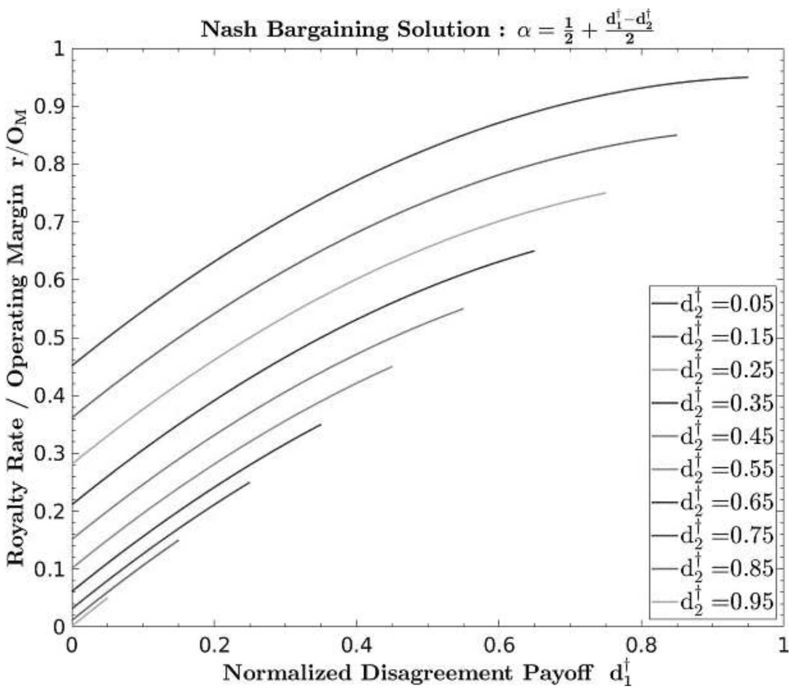

Figure 2: Family of Nash Bargaining Solutions for Table 1 Case 1

Note that a party is penalized to a much greater extent for having a weak disagreement payoff position over the original NBS of Fig. 1.

Case 2

In Case 2 of Table 1, each party assumes its bargaining strength is equal to its fraction of the total disagreement payoff position $d_1^\dagger + d_2^\dagger$. Moreover, each party agrees the other party's bargaining strength is its fraction of the total disagreement payoff. Substituting Case 2 of Table 1 into Eq. (12):

$$\frac{r}{O_M} = \frac{d_1^\dagger}{d_1^\dagger + d_2^\dagger} = \frac{1}{1 + d_2^\dagger / d_1^\dagger} \tag{23}$$

Interestingly, the payoff for each party is the party's bargaining weight. Moreover, the solution is independent of O_I, making this a non-cooperative bargain and equivalent to a limiting case of the Rubinstein model[36], where the parties take turns making an offer until an agreement is secured.[37]

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[36] Ariel Rubinstein, *Perfect Equilibrium in a Bargaining Model*, ECONOMETRICA: JOURNAL OF THE ECONOMETRIC SOCIETY 97-109 (1982); Binmore et al.,, *supra* note 14, at 182; MUTHOO, *supra* note 1118, at 46, 52 (noting the Subgame Perfect Equilibrium solution, where the time limit between offers $\Delta \rightarrow 0$, is presented in terms of discount rates (r_A, r_B) where $d_1^\dagger / d_2^\dagger = r_B / r_A$. The payoff pair obtained through perpetual disagreement, the Impasse Point, is $(\boldsymbol{I}_A, \boldsymbol{I}_B) = (d_1^\dagger, d_2^\dagger)$).

[37] MUTHOO, *supra* note 11, at 41, 47 (first discussing the Rubinstein model, where the parties take turns in making an offer until an agreement is secured: "[a]nother insight is that a party's bargaining power depends on the relative magnitude of the parties' respective costs of haggling, with the absolute magnitudes of these costs being irrelevant to the bargaining outcome. . . ." And then stating "[i]n a boxing match, the winner is the relatively stronger of the two boxers; the absolute strengths of the boxers are irrelevant to the outcome.")

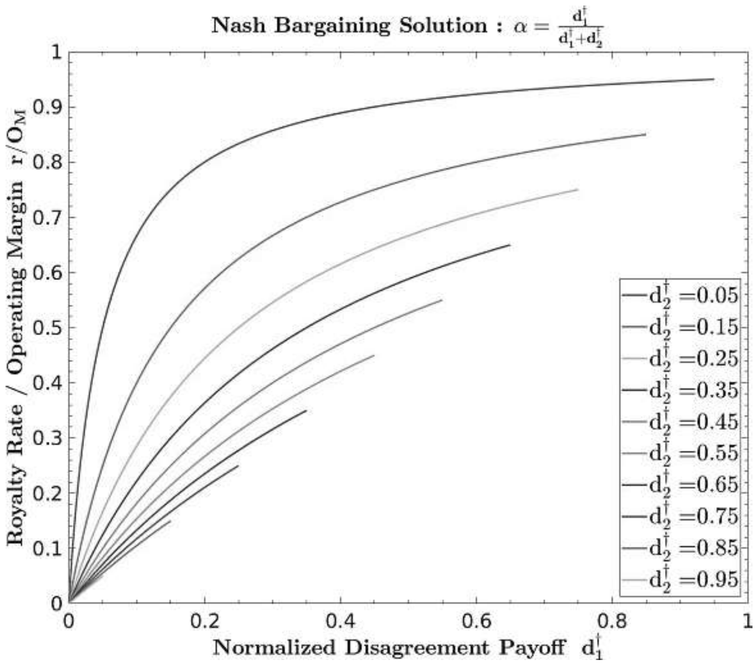

Figure 3: Family of Nash Bargaining Solutions for Table 1 Case 2

Fig. 3 shows the family of solutions for Eq. (23). Note the rapid collapse to zero of party 1's royalty for any constant $d_2^\dagger$ as $d_1^\dagger$ approaches zero.[38] The Rubinstein model has been used in recent court cases.[39]

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[38] *Cf.* United States v. AT& T, Inc., 310 F. Supp. 3d 161, 164 (D.D.C. 2018) (aff'd, 916 F.3d 1029 (D.C. Cir. 2019)) (a notable antitrust case that uses the NBS, showing in antitrust litigation, Case 1 or Case 2 could be used to set a threshold on the bargaining weight where one firm is shown to have significantly more bargaining power to trigger litigation. For example, if $\alpha \geq 0.75$ in Case 1, this could be a threshold for which litigation may be warranted).

[39] REBBECCA Reed-Arthurs et al., *Resolving Bargaining Range Indeterminacy in Patent Damages Patent Damages After VirnetX*, *in* THE LAW AND ECONOMICS OF PATENT DAMAGES, ANTITRUST, AND LEGAL PROCESS 7 (James Langenfelf et al. eds., 2021).

Case 3

Case 3 presents an example where party 2's bargaining strength depends on party 1's weakness. As in the previous examples, all parties agree on each other's bargaining strength. Substituting Case 3 of Table 1 into Eq. (12):

$$\frac{r}{O_M} = \frac{{d_2^\dagger}^2 - {d_1^\dagger}^2 - 2d_2^\dagger + d_1^\dagger + 1}{2 - d_1^\dagger - d_2^\dagger} \tag{24}$$

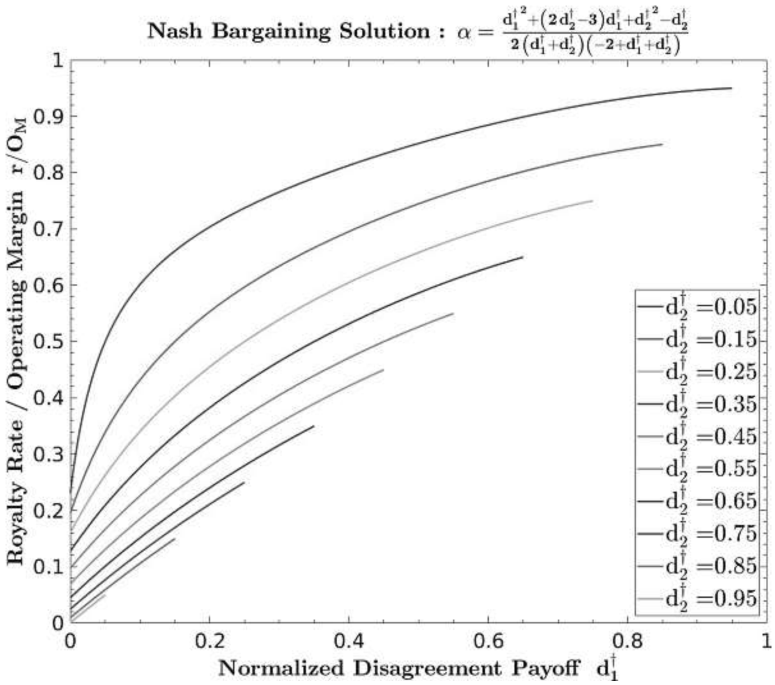

Figure 4: Family of Nash Bargaining Solutions for Table 1 Case 3

Fig. 4 shows the family of solutions for Eq. (24). The figure shows the same quadratic dependence as Case 1 Fig. 2, where the lines of constant $d_2^\dagger$ get closer together as $d_2^\dagger$ becomes dominant. Party 1's bargaining advantage has increased from Case 2 for small $d_1^\dagger$ because party 2's strength is derived from party 1's weakness and not its strength as in Case 2.

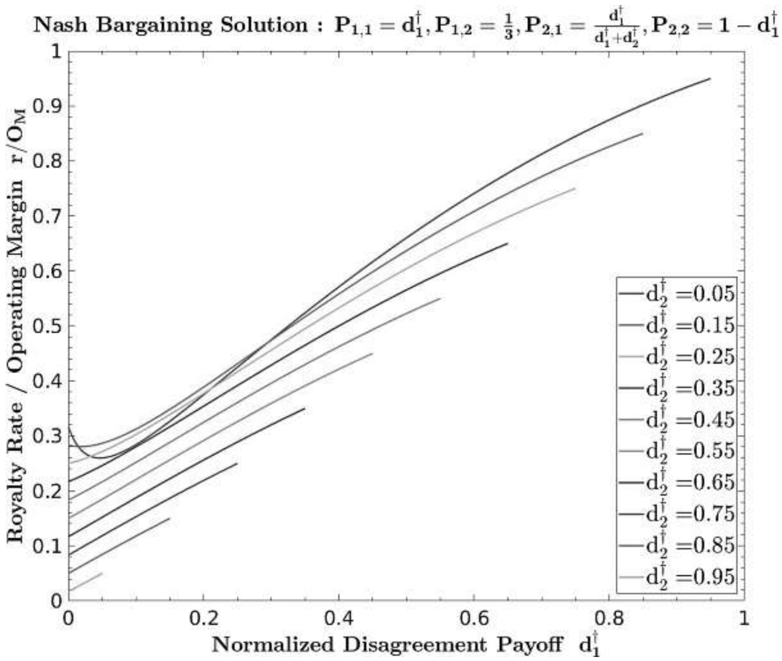

Figure 5: Family of Nash Bargaining Solutions With Regions That Violate Pareto Efficiency

C. Estimation of Bargaining Strength Using Combinations

Perceptions, independent or dependent of the disagreement payoffs, can be combined in Eq. (16). However, there are cases when combinations of perceptions are not Pareto efficient, which is examined next.

1. Solutions That Violate Pareto Efficiency

When α is a function of the disagreement payoffs, there can be combinations of perceptions that violate Pareto efficiency in a part of the solution space. Fig. 5 is one such example. Substituting the following hypothetical α into Eq. (12), Fig. 5 is obtained:

$$\alpha(d_1^\dagger, d_2^\dagger) = \frac{1}{2} + \frac{1}{4}\left[d_1^\dagger + \frac{1}{3} - \frac{d_1^\dagger}{d_1^\dagger + d_2^\dagger} - (1 - d_1^\dagger)\right] \quad (25)$$

From Fig. 5, it can be seen that the solution space is not Pareto efficient everywhere because when both $d_1^\dagger$ and $d_2^\dagger$ are small, party 1 will receive a lower royalty for a slight increase in $d_1^\dagger$, which is counterintuitive.

It is easily shown that the royalty in Fig. 5 violates Eq. (14) when $d_2^\dagger$ is small. The reason for this violation is that the specification of $P_{2,1}$ causes party 2's strength as perceived by party 1 to be lower as party 1's disagreement payoff lowers. This influences a small section of the solution space to violate Pareto efficiency.

VII. NOMOGRAPHS

To make it easy to compute a royalty using the asymmetric NBS, a nomograph was constructed (see Fig.6) with PyNomo.[40] A nomograph is a diagram that is a graphical representation of a mathematical function, and it allows for quick computation without substituting numbers into a formula.[41] Nomographs also provide

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[40] PyNomo, http://pynomo.org/wiki/index.php/Main_Page (last visited Apr. 8, 2022); www.pynomo.org, Nomographs with Python, http://www.myreckonings.com/modernnomograms/ (last visited Mar. 23, 2022).; www.myreckonings.com

[41] Leslie Glasser & Ron Doerfler, *A Brief Introduction to Nomography: Graphical Representation of Mathematical Relationships*, 50 Int'l J. Mathematical Educ. Sci. & Tech. 1273, 1273 (2018).

visualization of how the asymmetric NBS behaves so it can be easily explained.

To use the nomograph, pick any three variables on the graph and draw a straight line to get the fourth variable. For example, suppose that the normalized disagreement payoffs are $\mathrm{d}_1^\dagger = 0.20$ and $\mathrm{d}_2^\dagger = 0.30$. Additionally, suppose $\alpha = 0.40$. Using a straight edge, a line is drawn from $\alpha = 0.40$ to a point on the grid where$(\mathrm{d}_1^\dagger, \mathrm{d}_2^\dagger) = (0.20{,}0.30)$. The royalty for party 1 is read off the corresponding scale.

A blank nomograph is provided following the conclusion.

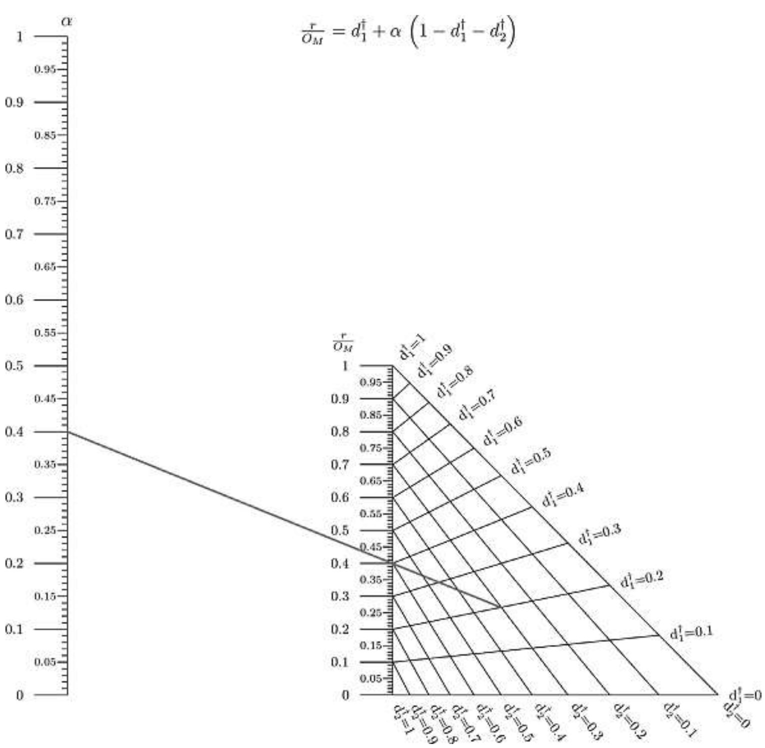

Figure 6: The use of the nomograph is demonstrated with $d^\dagger{}_1 = 0.20$, $d^\dagger{}_2 = 0.30$, and $\alpha = 0.40$ to solve for $r/O_M = 0.40$.

VIII. CONCLUSION

In this model of the asymmetric NBS, there are three essential variables needed to obtain a royalty. They are the normalized disagreement payoffs of both party 1 and party 2, and the bargaining weight. At a minimum, the parties should have a good understanding of the licensed product's operating margin if a royalty rate is to be computed along with the need to make educated guesses on the normalized disagreement payoffs of both parties. Various examples were given to demonstrate how each party's bargaining strengths can be incorporated into the bargaining weight. These individual bargaining strengths can be used to apply the NBS to the specific facts of the case. Although *Georgia-Pacific* factor fifteen is the basis for this analysis, the other fourteen factors could also be used to obtain the normalized disagreement payoffs and choose the bargaining strengths. Finally, a nomograph has been produced so the parties can easily calculate the asymmetric NBS and solve for a reasonable royalty.

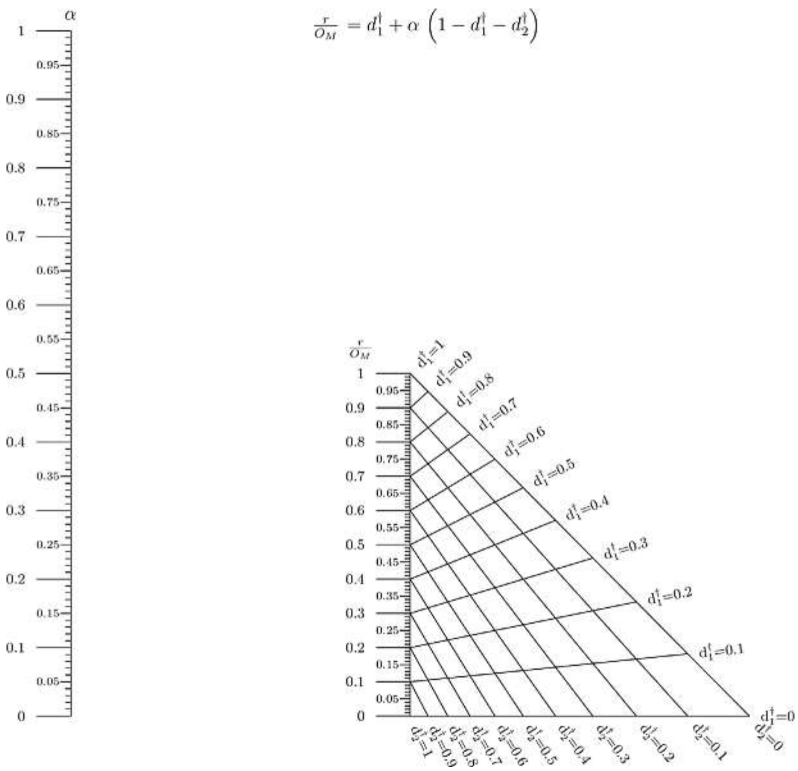

Figure 7: Blank Nomograph